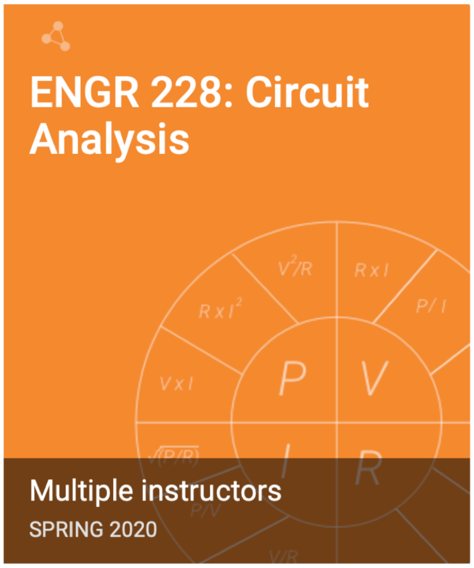


Chapter 8.4-5

Complex Power and Power Factor

Engr228 - Circuit Analysis

Spring 2020

Dr Curtis Nelson

Sections 8.4-5 Objectives

- Determine the complex power, average power, and reactive power for any complex load with known input voltage or current.
- Determine the power factor for a complex load and evaluate the improvement realized by compensating the load through the addition of a shunt capacitor.

Instantaneous vs. Average Power

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

The equation above can be simplified as follows:

$$p(t) = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

where

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) \qquad \textit{(Average Power)}$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) \qquad \textit{(Reactive Power)}$$

Effective or RMS Value

- For example, the effective value of $I = I_m \cos\omega t$ is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t dt}$$

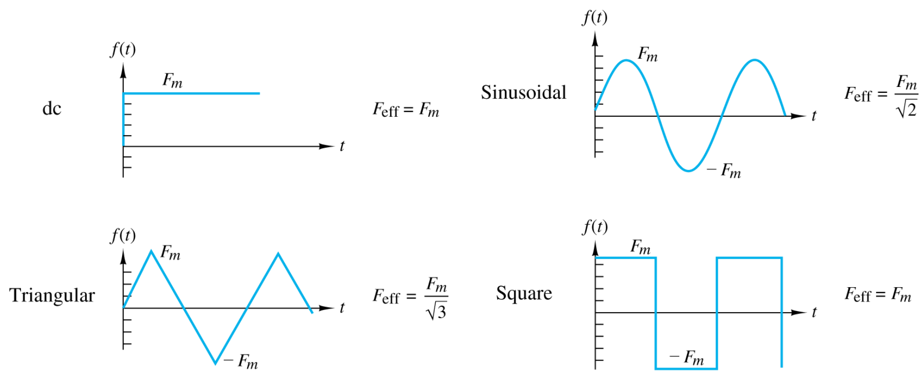
$$I_{rms} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t) dt} = \frac{I_m}{\sqrt{2}}$$

- Average power can then be written in two ways:

$$\boxed{P_{AVG} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}$$

$$\boxed{P_{AVG} = V_{rms} I_{rms}\cos(\theta_v - \theta_i)}$$

Effective Values of Common Waveforms

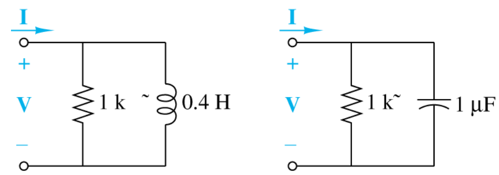


The Angle θ_V - θ_i

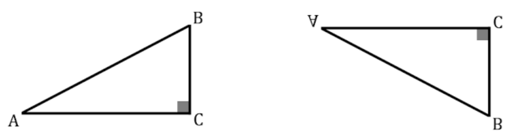
- $\theta_V - \theta_i$ is known as the ***Power Factor Angle.***
- $\cos(\theta_V - \theta_i)$ is known as the ***Power Factor (PF)***
  - For a purely resistive load, *PF=1;*
  - For a purely reactive load, *PF=0;*
  - For any practical circuit, $0 \le PF \le 1$.
- We must distinguish between positive and negative arguments for the power factor since cos(x) = cos(-x).

Leading and Lagging Power Factor

- pf is ***lagging*** if the current lags the voltage (θ_V - θ_I is positive like in an inductive load);
- pf is ***leading*** if the current leads the voltage (θ_V - θ_I is negative like in a capacitive load).

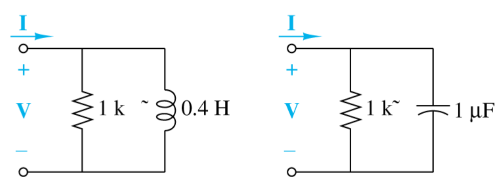


(a) (b)



Power Factor - Example

- Calculate the power factor of each circuit below if the applied voltage is V = *10cos(2500 t)* V



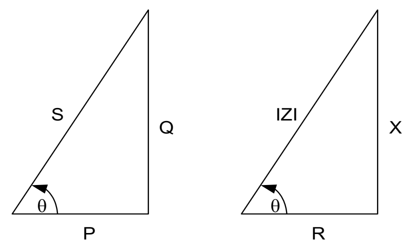
(a) (b)

pf_{ind} = *0.707 lagging*

pf_{cap} = *0.371 leading*

Power Factor and the Impedance Triangle

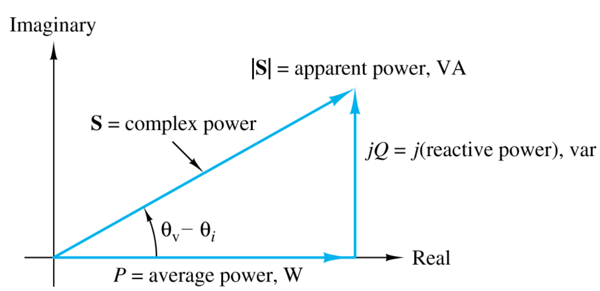
The **Power Factor** is also defined as:



Power triangle   Impedance triangle

$$pf = \cos\theta = \frac{P}{S} = \frac{R}{|Z|}$$

Complex Power - Triangle



Complex power contains both a real part (**average power**) and an imaginary part (**reactive power**) that together are represented in a *power triangle*. In equation form:

$$\boldsymbol{S = P + jQ}$$

Complex Power - Units

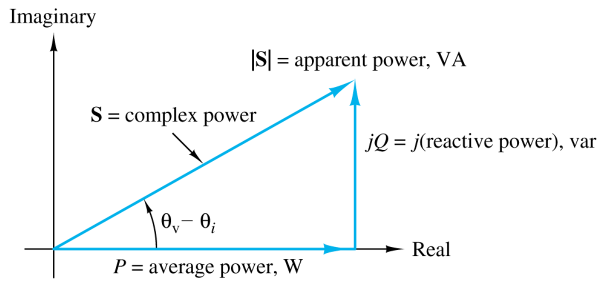
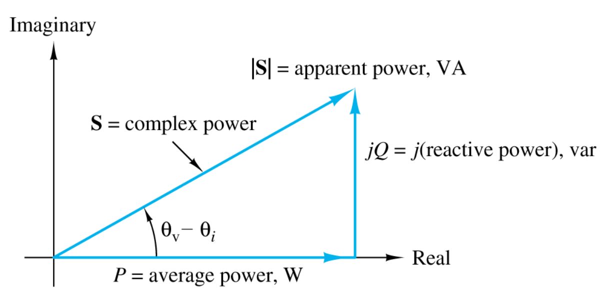


TABLE 10.2 Three Power Quantities and Their Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

Complex Power – The Equations

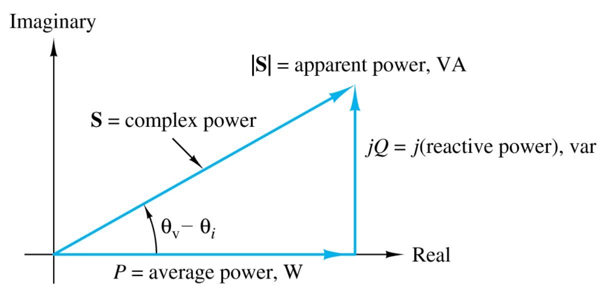


$P_{AVG} = V_{RMS} I_{RMS} \cos(\theta_V - \theta_i)$ *watts*

$P_{REACTIVE} = V_{RMS} I_{RMS} \sin(\theta_V - \theta_i)$ *volt-amps-reactive*

$P_{APPARENT} = V_{RMS} I_{RMS}$ *volt-amps*

Apparent Power and Power Factor

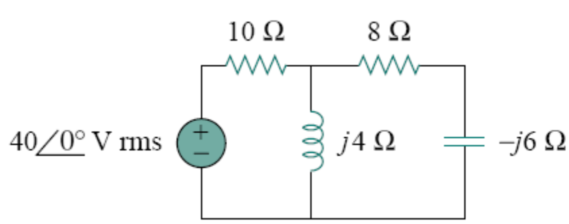


Power factor can also be defined as:

$$PF = \frac{average\ power}{apparent\ power} = \frac{P_{avg}}{V_{rms} I_{rms}}$$

Example - Power Factor

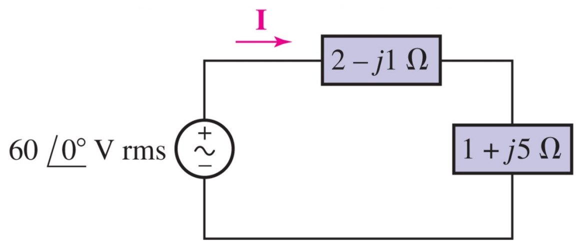
Calculate the power factor of the circuit below as seen by the source. What is the average power supplied by the source?



pf = *0.936 (lagging)*
$P_{AVG} = 118W$

Example Problem

Find the average power delivered to each of the two loads, the apparent power supplied by the source, and the power factor of the combined loads.



$P_{avg2} = 288\ W$

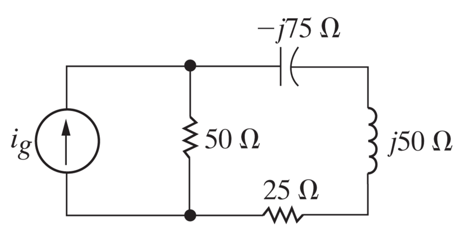
$P_{avg1} = 144\ W$

$P_{app} = S = 720\ VA$

$PF = 0.6$ *(lagging)*

Textbook Problem 10.17 - Nilsson 10th

For $i_g = 50 + j0$ mA (rms), find the average and reactive power for the current source and state whether it is absorbing or delivering this power.

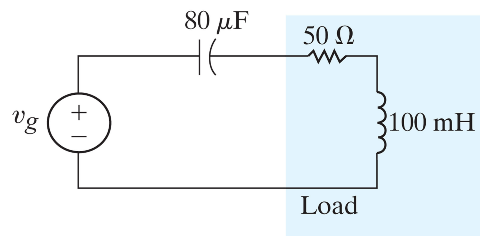


$P_{AVG} = -50.0\ mW$ *(delivering)*

$P_{REAC} = 25.0\ mVAR$ *(absorbing)*

Textbook Problem 10.20 - Nilsson 11th

Find the average power, the reactive power, and the apparent power absorbed by the load if v_g = 150cos250t V.

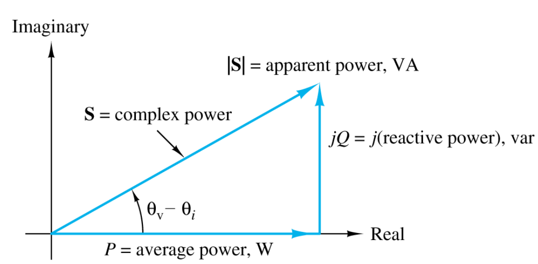


P_{AVG} = 180 W

P_{REAC} = 90 VAR

P_{APP} = 180 + j90 VA

Alternative Forms for Complex Power



$$S = P + jQ$$
$$= V_{rms}I_{rms}\angle\theta_V - \theta_I$$
$$= V_{rms}\angle\theta_V\, I_{rms}\angle{-\theta_I}$$
$$= V_{rms}I^*_{rms}$$

Complex Power - Summary

Real (Average) power:

$$P = \mathrm{Re}(S) = S\cos(\theta_v - \theta_i) \quad \textbf{Watts}$$

Reactive power:

$$Q = \mathrm{Im}(S) = S\sin(\theta_v - \theta_i) \quad \textbf{VAR}$$

Apparent power:

$$S = |S| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2} \quad \textbf{VA}$$

Complex power:

$$S = P + jQ = V_{RMS} I^*{}_{RMS} \quad \textbf{VA}$$

- Q = 0 for resistive loads (unity power factor);
- Q < 0 for capacitive loads (leading power factor);
- Q > 0 for inductive loads (lagging power factor).

Example - Apparent Power and Power Factor

- A series connected RC load draws a current of

  $$i(t) = 4\cos(100\pi t + 10^\circ)A$$

  when the applied voltage is

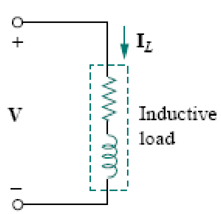
  $$v(t) = 120\cos(100\pi t - 20^\circ)V$$

- Find the apparent power and the power factor of the load and determine the element values that form the load.

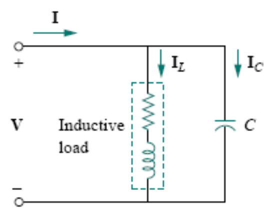
S = 240 V
pf = 0.866 (leading)
C = 212.2μF
R = 26.0Ω

Power Factor Correction

- The process of increasing the power factor without altering the voltage or current to the original load is known as **power factor correction.**
- Most loads are inductive. The power factor of the load can be improved (to make it closer to unity) by installing a capacitor in parallel with an inductive load.

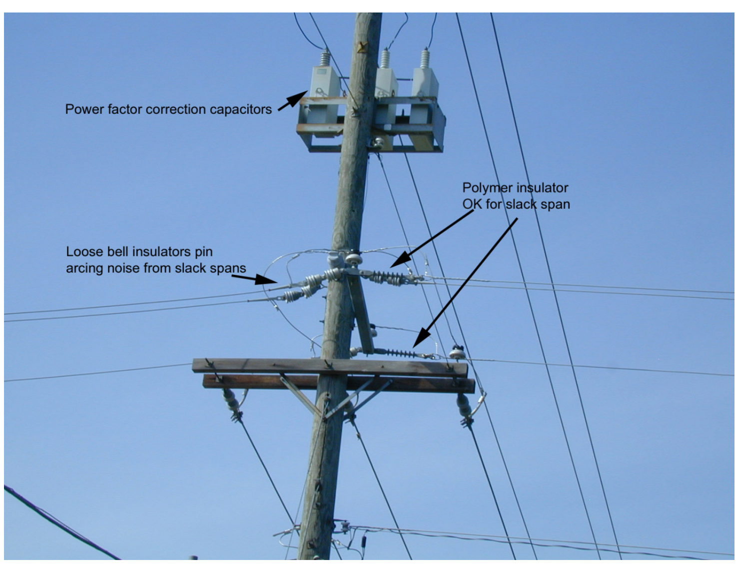


a) Original inductive load



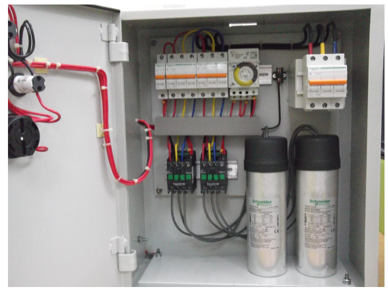
b) Inductive load with improved power factor

Power Factor Correction Capacitors



Power Factor Correction Capacitors

Reduce your Electricity Bill
and improve your Profitability



Power Factor Correction by Power Triangle

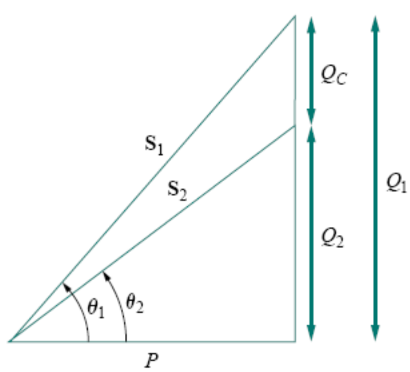
The triangle below shows that the reactive power (Q_1) in a large inductive load can be reduced by an amount (Q_C) by adding a capacitor in parallel to the inductive load.

$$Q_1 = P\tan\theta_1$$

$$Q_2 = P\tan\theta_2$$

$$Q_C = Q_1 - Q_2$$

$$= P(\tan\theta_1 - \tan\theta_2)$$



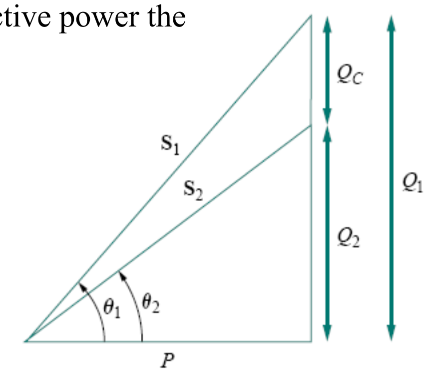
Power Factor Correction by Power Triangle

- But Q_C is also the amount of reactive power the capacitor must provide:

$$Q_C = \frac{V_{rms}^2}{X_C} = \omega C V_{rms}^2$$

- The value of required parallel capacitance is then:

$$\boxed{C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}}$$



Example - PF Correction

Find the value of parallel capacitance needed to correct a load of 140kVAR at 0.85 lagging pf to unity pf. Assume that the load is supplied by a $110V_{RMS}$, 60Hz line.

$$\boxed{C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}}$$

C = 30,691μF

Textbook Problem 10.31 - Nilsson 11th

A group of small appliances on a 60 Hz system requires 20 kVA at 0.85 pf lagging when operated at 125 V_{rms}. The impedance of the feeder supplying the appliances is 0.01+ j0.08 Ω. Find:

a) The rms value of the voltage at the source end of the feeder.

b) The average power loss in the feeder.

c) What size capacitor at the load end of the feeder is needed to improve the load power factor to unity?

$V_S = 133.48\ V_{RMS}$

$P_{REAC} = 256\ W$

$C = 1188.7\ \mu F$

Power Generation

Hydro-Power Generation

Solar Power

Nuclear Power Generation

Fossil Fuel Generation

Wave Power Generation

Gravity Light Project

Sections 8.4-5 Summary

From the study of this section, you should be able to:

- Determine the complex power, average power, and reactive power for any complex load with known input voltage or current.
- Determine the power factor for a complex load and evaluate the improvement realized by compensating the load through the addition of a shunt capacitor.